

Technical note

Thermodynamic analysis of a Stirling engine including dead volumes of hot space, cold space and regenerator

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Abstract

This paper provides a theoretical investigation on the thermodynamic analysis of a Stirling engine. An isothermal model is developed for an imperfect regeneration Stirling engine with dead volumes of hot space, cold space and regenerator that the regenerator effective temperature is an arithmetic mean of the heater and cooler temperature. Numerical simulation is performed and the effects of the regenerator effectiveness and dead volumes are studied. Results from this study indicate that the engine net work is affected by only the dead volumes while the heat input and engine efficiency are affected by both the regenerator effectiveness and dead volumes. The engine net work decreases with increasing dead volume. The heat input increases with increasing dead volume and decreasing regenerator effectiveness. The engine efficiency decreases with increasing dead volume and decreasing regenerator effectiveness.

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Nomenclature

C_V	specific heat at constant volume (J/kg K)
e	regenerator effectiveness
E_S	Stirling engine thermal efficiency
K	factor defined by Eq. (9)
k	specific heat ratio
k_{SH}	V_{SH}/V_S is hot space dead volume ratio
k_{SR}	V_{SR}/V_S is regenerator dead volume ratio
k_{SC}	V_{SC}/V_S is cold space dead volume ratio
k_{SDP}	$V_S/(V_D + V_P)$ is total dead volume to total volume ratio
k_{ST}	V_S/V_1 is total dead volume to total volume ratio
m	total working fluid mass contained in the engine (kg)
p	absolute pressure (N/m ²)
p_m	cycle mean effective pressure (N/m ²)
Q_{in}	total heat added from an external source to the cycle (J)
Q_{out}	total heat rejected from the cycle to an external sink (J)
R	gas constant (J/kg K)
T_3	working fluid temperature in the hot space (K)
$T_{3'}$	working fluid temperature at regenerator outlet (K)
T_1	working fluid temperature in the cold space (K)
$T_{1'}$	working fluid temperature at regenerator inlet (K)
T_R	effective working fluid temperature in regenerator dead space (K)
T_C	cooler temperature (K)
T_H	heater temperature (K)
V_{SH}	hot-space dead volume (m ³)
V_{SR}	regenerator dead volume m ³
V_{SC}	cold-space dead volume (m ³)
V_S	total dead volume (m ³)
V_D	displacer swept volume (m ³)
V_P	power piston swept volume (m ³)
W_{net}	engine network (J)

1. Introduction

The Stirling engine is a simple type of external-combustion engine that uses a compressible fluid as a working fluid. The Stirling engine can theoretically be a very efficient engine to convert heat into mechanical work at Carnot efficiency. The thermal limit for the operation of a Stirling engine depends on the material used for its construction. In most instances, the engines operate with a heater and cooler temperature of 923 and 338 K, respectively [1]. Engine efficiency ranges from about 30 to 40% resulting from a typical temperature range of 923–1073 K, and normal operating speed range from 2000 to 4000 rpm [2].

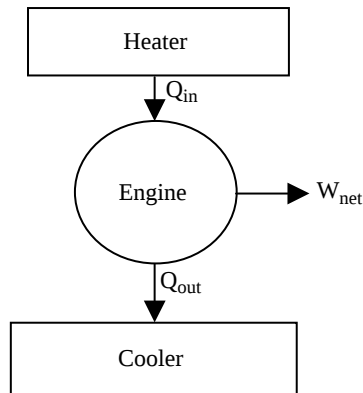
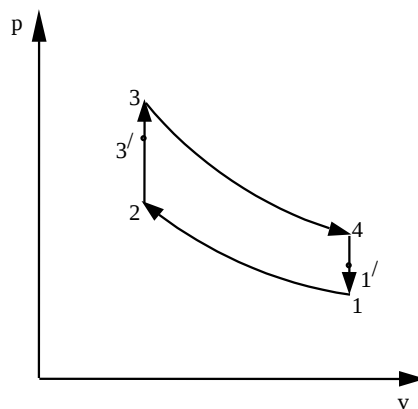


Fig. 1. Schematic diagram.

Fig. 1 shows a simplified schematic diagram of an imperfect-regeneration Stirling engine. The p – v diagram for the Stirling cycle with imperfect regeneration is shown in Fig. 2. For an ideal regeneration, the total heat rejected during process 4–1 is absorbed by a perfect regenerator and released to the working fluid during process 2–3 since in the ideal regeneration an infinite heat-transfer area or an infinite regeneration time is needed.

For an imperfect regenerator, the working fluid temperature at the regenerator outlet and inlet will be $T_{3'}$ and $T_{1'}$, respectively. An external heat input and output is required to increase $T_{3'}$ to T_3 and decrease $T_{1'}$ to T_1 [1]. Although regenerator effectiveness values of 95, 98–99, and 99.09% have been reported [3–7], engine developers who do not have efficient-regenerator technology in hand should take into account the regenerator effectiveness, and then an analysis with imperfect regeneration should be made.

Dead volume is defined as the total void volume in a Stirling engine. In general, the dead volume is referred to as the volume of working fluid contained in the total dead space in the engine, including the regenerator and transfer port. It is evidenced that a real Stirling

Fig. 2. p – v Diagram.

engine must have some unavoidable dead volume. In normal Stirling engine design practice, the total dead volume is approximately 58% of the total volume [8].

Although many researchers have analyzed Stirling engines, there still remains room for further development. Investigation on Stirling engine analysis [9] shows that almost all literature deals with ideal regeneration and zero dead volume. The imperfect regeneration Stirling engine including dead volumes has received comparatively little attention in literature and should be studied in more detail.

The Stirling engine including dead volume can be analyzed by the Schmidt technique [10]. However, ideal regeneration is assumed in the Schmidt analysis [1,11]. For Stirling engines with large dead volumes, having the correct working fluid temperature in the regenerator is important. The effective temperature of the working fluid contained in the regenerator dead space can be calculated by separating the dead volume into two [8,11], one half is at $T_{3'}$ and the other half is at $T_{1'}$. The regenerator effective temperature in another ways is the arithmetic mean [1,8,11] and the log mean temperatures [8,11] of working fluid at the outlet and inlet of regenerator.

Martini [8] used the log-mean regenerator effective temperature in the analysis of a Stirling engine with dead volume. The log-mean regenerator effective temperature was found to be infinite at 50% regenerator effectiveness, since $T_{3'}$ equals $T_{1'}$. Therefore, the log-mean regenerator effective temperature should not be used with variable regenerator effectiveness. However, the definition of the regenerator effectiveness used by Martini [8] is different from the definition commonly used by others [1,11]. The arithmetic-mean regenerator effective temperature should give good approximation [11].

The objective of this article is to analyze an imperfect regeneration Stirling engine with dead volume, based on classical thermodynamics. The analysis is in a general form that takes into account both the regenerator effectiveness and the dead volume. The results obtained in this article will provide a generalized guideline for the analysis of a Stirling engine with imperfect regeneration and dead volume.

2. Stirling cycle analysis

2.1. Dead volumes

Assume that the hot space, regenerator and cold space dead volumes, in m^3 , are respectively V_{SH} , V_{SR} and V_{SC} , then, the total dead volume is

$$V_{\text{S}} = V_{\text{SH}} + V_{\text{SR}} + V_{\text{SC}} = (k_{\text{SH}} + k_{\text{SR}} + k_{\text{SC}})V_{\text{S}} \quad (1)$$

where $k_{\text{SH}} = V_{\text{SH}}/V_{\text{S}}$ is the hot space dead volume ratio, $k_{\text{SR}} = V_{\text{SR}}/V_{\text{S}}$ is the regenerator dead volume ratio and $k_{\text{SC}} = V_{\text{SC}}/V_{\text{S}}$ is the cold space dead volume ratio.

Let the total dead volume to total volume ratio be represented as $k_{\text{ST}} = V_{\text{S}}/V_1$, then the total dead volume can be expressed in terms of total volume as

$$V_{\text{S}} = k_{\text{ST}}V_1 = k_{\text{ST}}(V_{\text{S}} + V_{\text{D}} + V_{\text{P}}) \quad (2)$$

where V_D and V_P are the displacer and power piston swept volumes in m^3 , respectively. The dead volume is more conveniently expressed in terms of the total swept volume as:

$$V_S = k_{SDP}(V_D + V_P) \quad (3)$$

Therefore, the dead volume to the total volume ratio, and the dead volume to the total swept volume ratio are related by:

$$k_{ST} = k_{SDP}/(1 + k_{SDP}) \text{ or } k_{SDP} = k_{ST}/(1 - k_{ST}) \quad (4)$$

2.2. Imperfect regenerator

The regenerator effectiveness, e , of an imperfect regenerator is defined as [1,8,9]:

$$e = (T_{3'} - T_1)/(T_3 - T_1) \quad (5)$$

The value of e is 1 for 100% effectiveness or ideal regeneration and e is 0 for 0% effectiveness or no regeneration. The working fluid temperature at the regenerator outlet can be expressed in terms of the regenerator effectiveness as:

$$T_{3'} = T_1 + e(T_3 - T_1) \quad (5a)$$

For a regenerator having equal effectiveness in heating and cooling, $Q_{2-3'} = Q_{4-1'}$, the working fluid temperature at the regenerator inlet is:

$$T_{1'} = T_3 + e(T_1 - T_3) = T_3 - e(T_3 - T_1) \quad (6)$$

For the Stirling engines with large dead volumes, having the correct working fluid temperature for the regenerator is important. The effective temperature of the working fluid contained in the regenerator dead space can be determined using the arithmetic mean [1,8,11]:

$$T_R = (T_{3'} + T_{1'})/2 \quad (7)$$

Substitute Eqs. (5a) and (6) into Eq. (7) gives:

$$T_R = (T_3 + T_1)/2 \quad (7a)$$

It can be seen that by using the arithmetic mean the regenerator effective temperature is not dependent on the regenerator effectiveness.

2.3. Equation of state

Assume that the hot space and cold space volumes are, respectively, V_H and V_C and that the working fluid temperatures in the hot space, regenerator, and cold space are, respectively, T_3 , T_R and T_1 . The equation of state for the isothermal compression process 1-2 with dead volumes V_{SH} , V_{SR} and V_{SC} is [8]

$$p = mR/(V_H/T_3 + V_{SH}/T_3 + V_{SR}/T_R + V_{SC}/T_1 + V_C/T_1) = mR/(V_H/T_3 + K + V_C/T_1) \quad (8)$$

where

$$K = V_{SH}/T_3 + V_{SR}/T_R + V_{SC}/T_1 \quad (9)$$

and m is the total working fluid mass contained in the engine in kg. Substitute Eqs. (1) and (7a) into Eq. (9) gives:

$$K = \left(\frac{k_{SH}}{T_3} + \frac{2k_{SR}}{T_3 + T_1} + \frac{k_{SC}}{T_1} \right) V_S \quad (9a)$$

It is clear that, for a given hot-side and cold-side working fluid temperature, the factor K in general is a function of the dead volumes.

2.4. Isothermal compression process

In the compression process, the cold-side working fluid is compressed from $V_{C1} = V_D + V_P$ to $V_{C2} = V_D$. The hot-space working fluid swept volume, V_H , is 0 throughout this process. Then the heat rejected during the isothermal compression process 1-2 is:

$$\begin{aligned} Q_{1-2} &= W_{1-2} = \int_{V_{C1}}^{V_{C2}} p \, dV_C = mRT_1 \int_{V_{C1}}^{V_{C2}} dV_C / (V_C + KT_1) \\ &= mRT_1 \ln[(V_{C2} + KT_1)/(V_{C1} + KT_1)] \\ &= mRT_1 \ln[(V_D + KT_1)/(V_D + V_P + KT_1)] \end{aligned} \quad (10)$$

It should be noted that the compression work depends only on the factor K , which is a function of the dead volumes.

2.5. Isochoric heating process

In principle, the heat added during the isochoric heating process 2-3 is:

$$Q_{2-3} = mC_V(T_3 - T_2) = mC_V(T_3 - T_1) \quad (11)$$

where C_V is the specific heat at constant volume in J/kg K, and is assumed to be constant. Without regeneration, this amount of heat is added by an external source and for ideal regeneration this amount of heat is released from an ideal regenerator.

The regeneration heat released from an imperfect regenerator during this process is:

$$Q_{2-3'} = mC_V(T_{3'} - T_2) = emC_V(T_3 - T_1) \quad (12)$$

Heat added from an external source during process 3'-3 is:

$$Q_{3-3'} = mC_V(T_3 - T_{3'}) = (1 - e)mC_V(T_3 - T_1) \quad (13)$$

It can be seen that the heat input to this process depends on the regenerator effectiveness only.

2.6. Isothermal expansion process

In the expansion process, the hot-side working fluid volume changes from $V_{H3} = V_D$ to $V_{H4} = V_D + V_P$. The cold-space working fluid swept volume, V_C , is 0 throughout this process. The heat added to the cycle during the isothermal expansion process 3-4 is:

$$\begin{aligned} Q_{3-4} &= W_{3-4} = m \int_{V_{H3}}^{V_{H4}} p \, dV_H = mRT_3 \int_{V_{H3}}^{V_{H4}} dV_H / (V_H + KT_3) \\ &= mRT_3 \ln[(V_{H4} + KT_3)/(V_{H3} + KT_3)] \\ &= mRT_3 \ln[(V_D + V_P + KT_3)/(V_D + KT_3)] \end{aligned} \quad (14)$$

It is evidenced that the expansion work is dependent on the dead volumes.

2.7. Isochoric cooling process

The heat rejected during the isochoric cooling process 4-1 is:

$$Q_{4-1} = mC_V(T_1 - T_4) = -mC_V(T_3 - T_1) \quad (15)$$

Without regeneration, this amount of heat is rejected to an external sink and for ideal regeneration, an ideal regenerator absorbs this amount of heat.

For an imperfect regeneration, the heat absorbed by a regenerator is:

$$Q_{4-1'} = mC_V(T_{1'} - T_4) = -emC_V(T_3 - T_1) \quad (16)$$

The heat rejected to an external sink during process 1'-1 is:

$$Q_{1-1'} = mC_V(T_1 - T_{1'}) = -(1 - e)mC_V(T_3 - T_1) \quad (17)$$

It can be seen that the heat transfer in the cooling process depends only on the regenerator effectiveness.

2.8. Total heat added

For an imperfect regeneration, the total heat added from an external source to the cycle is:

$$Q_{in} = Q_{3-3'} + Q_{3-4} \quad (18)$$

$$\begin{aligned} Q_{in} &= mC_V\{(T_3 - T_{3'}) + (k - 1)T_3 \ln[(V_{H4} + KT_3)/(V_{H3} + KT_3)]\} \\ &= mC_V\{(1 - e)(T_3 - T_1) + (k - 1)T_3 \ln[(V_D + V_P + KT_3)/(V_D + KT_3)]\} \end{aligned} \quad (18a)$$

where k is the specific heat ratio. Therefore, the heat input to the engine depends on both the regenerator effectiveness and dead volumes.

Without regeneration, the total heat added from an external source is:

$$Q_{in} = Q_{2-3} + Q_{3-4} \quad (19)$$

However, for an ideal regeneration, the total heat added from an external source will be:

$$Q_{\text{in}} = Q_{3-4} \quad (20)$$

2.9. Total heat rejected

For an imperfect regeneration, the total heat rejected from the cycle to an external sink is:

$$Q_{\text{out}} = Q_{1-1'} + Q_{1-2} \quad (21)$$

$$\begin{aligned} Q_{\text{out}} &= -mC_V\{(T_1 - T_{1'}) + (k - 1)T_1 \ln[(V_{C1} + KT_1)/(V_{C2} + KT_1)]\} \\ &= -mC_V\{(1 - e)(T_3 - T_1) + (k - 1)T_1 \ln[(V_D + V_P + KT_1)/(V_D + KT_1)]\} \end{aligned} \quad (21a)$$

The heat rejected from the engine also depends on both the dead volumes and regenerator effectiveness.

Without regeneration, the total heat rejected to an external sink is:

$$Q_{\text{out}} = Q_{4-1} + Q_{1-2} \quad (22)$$

For an ideal regeneration, the total heat rejected to an external sink can be only:

$$Q_{\text{out}} = Q_{1-2} \quad (23)$$

It is evidenced that the amounts of heat added to the cycle and rejected from the cycle are dependent on the regeneration heating and cooling.

2.10. Net work

The surplus energy of the two isothermal processes 1-2 and 3-4 is converted into useful mechanical work. The net work for an imperfect-regeneration engine with dead volumes can be determined from:

$$\begin{aligned} W_{\text{net}} &= \sum Q = Q_{\text{in}} - Q_{\text{out}} = Q_{3-3'} + Q_{3-4} + Q_{1-1'} + Q_{1-2} = Q_{3-4} + Q_{1-2} \\ &= mR\{T_3 \ln[(V_{H4} + KT_3)/(V_{H3} + KT_3)] - T_1 \ln[(V_{C1} + KT_1)/(V_{C2} + KT_1)]\} \end{aligned} \quad (24)$$

Note that $V_{H4} = V_{C1} = V_D + V_P = V_1$ and $V_{H3} = V_{C2} = V_D = V_2$, therefore:

$$W_{\text{net}} = mR\{T_3 \ln[(V_D + V_P + KT_3)/(V_D + KT_3)] - T_1 \ln[(V_D + V_P + KT_1)/(V_D + KT_1)]\} \quad (24a)$$

It is evidenced that the cycle net work depends on the dead volumes.

In a case of zero dead volume, the cycle net work is:

$$W_{\text{net}} = mR \ln(V_1/V_2)(T_3 - T_1) \quad (25)$$

Eq. (25) can be found in many textbooks on thermodynamics. It should be noted that in the case of zero dead volume, the cycle net work is not dependent on the regenerator effectiveness.

2.11. Mean effective pressure

The engine net work can be determined from the cycle mean effective pressure, p_m , and total volume changed, $V_{H4} - V_{H3} = V_{C1} - V_{C2} = V_1 - V_2 = V_P$:

$$W_{\text{net}} = p_m V_P \quad (26)$$

Equate Eq. (26) to Eq. (24a), then:

$$p_m = (mR/V_P)\{T_3 \ln[(V_D + V_P + KT_3)/(V_D + KT_3)] - T_1 \ln[(V_D + V_P + KT_1)/(V_D + KT_1)]\} \quad (27)$$

It should be noted that the mean effective pressure depends on the dead volumes. For zero dead volume, by using the perfect gas law and noting that $T_1 = T_2$ and $V_2 = V_3$:

$$p_m = [(p_3 - p_2)V_3/(V_1 - V_2)] \ln(V_1/V_2) = [(p_3 - p_2)/(V_1/V_2 - 1)] \ln(V_1/V_2) \quad (28)$$

2.12. Thermal efficiency

The Stirling engine thermal efficiency can be determined from:

$$E_S = W_{\text{net}}/Q_{\text{in}} \quad (29)$$

$$E_S = \{T_3 \ln[(V_D + V_P + KT_3)/(V_D + KT_3)] - T_1 \ln[(V_D + V_P + KT_1)/(V_D + KT_1)]\} / \{T_3 \ln[(V_D + V_P + KT_3)/(V_D + KT_3)] + (T_3 - T_1)(1 - e)/(k - 1)\} \quad (29a)$$

It can be seen that the Stirling engine efficiency also depends on both the regenerator effectiveness and dead volume.

In the case of zero dead volume, Eq. (29a) will be reduced to:

$$E_S = (T_3 - T_1)/[T_3 + (T_3 - T_1)/(k - 1) \ln(V_1/V_2)] \quad (30)$$

Even when the different notation is used as in Eq. (30), the results are the same as in former works approached by classical thermodynamics [8,11,12] and finite-time thermodynamics [13]. Without regeneration, $e = 0$, which is the worst case of Stirling cycle efficiency, the thermal efficiency is:

$$E_S = (T_3 - T_1)/[T_3 + (T_3 - T_1)/(k - 1) \ln(V_1/V_2)] \quad (31)$$

For an ideal regeneration, $e = 1$, which is the best case of Stirling cycle efficiency, the thermal efficiency is:

$$E_S = (T_3 - T_1)/T_3 \quad (32)$$

This is the Carnot efficiency of an engine operating at the source and sink temperature of T_3 and T_1 , respectively.

It is evidenced that, in theory, the Stirling engine can be a very efficient device for converting heat into mechanical work requiring high-temperature difference. The efficiency of the Stirling cycle with ideal regeneration and zero dead volume equals that of the Carnot cycle, the most efficient of all ideal thermodynamic cycles. However, different from the Carnot cycle, the Stirling-cycle engine is a practical engine that can actually be used to produce useful work.

3. Solution method

In order to examine the effect of dead volume on engine performance, the equations for engine performance should be expressed in terms of the regenerator effectiveness and dead volumes. A non-pressurized air engine, with a conventional compression ratio of $CR = V_D/V_P = 1.5$, is used to as a numerical example. The initial principal engine design parameters given are:

Hot space temperature $T_3 = 923$ K
 Cold space temperature $T_1 = 338$ K
 Power piston swept volume $V_P = 50,000$ cc
 Displacer swept volume $V_D = 1.5$, $V_P = 75,000$ cc
 Hot-space dead volume ratio $k_{SH} = 0.2$
 Regenerator dead volume ratio $k_{SR} = 0.6$
 Cold-space dead volume ratio $k_{SC} = 0.2$

Computation procedure for an imperfect regeneration Stirling engine with a given regenerator effectiveness and total dead volume can be described as follows:

- (1) From Eq. (7a), for the working fluid temperature maintained at $T_3 = 923$ K and $T_1 = 338$ K, the effective regenerator temperature is:

$$T_R = (T_3 + T_1)/2 = 630.5 \text{ K}$$

- (2) Working cylinder volumes are:

$$V_{C1} = V_{H4} = V_D + V_P = 75,000 + 50,000 = 125,000 \text{ cc} = 0.125 \text{ m}^3$$

$$V_{C2} = V_{H3} = V_D = 75,000 \text{ cc} = 0.075 \text{ m}^3$$

From Eq. (4), dead volume to total swept volume ratio is

$$k_{SDP} = V_S/(V_D + V_P) = V_S/(0.125) \quad (33)$$

and dead volume to total volume is:

$$k_{ST} = k_{SDP}/(1 + k_{SDP}) = [V_S/(0.125)]/[1 + V_S/(0.125)] \quad (34)$$

From Eq. (1), dead volumes in hot space, regenerator and cold space are:

$$V_{SH} + V_{SR} + V_{SC} = (k_{SH} + k_{SR} + k_{SC})V_S = (0.2 + 0.6 + 0.2)V_S \text{ m}^3 \quad (35)$$

(3) From Eq. (9), the factor K is:

$$\begin{aligned} K &= \left(\frac{k_{SH}}{T_3} + \frac{k_{SR}}{T_R} + \frac{k_{SC}}{T_1} \right) V_S \\ &= [0.2/923 + (0.6)/630.5 + 0.2/338]V_S \text{ m}^3/\text{K} = 1.76 \times 10^{-3} V_S \text{ m}^3/\text{K} \end{aligned} \quad (36)$$

The products of factor K and heater and cooler temperature are:

$$KT_1 = 338\{1.76 \times 10^{-3}\}V_S \text{ m}^3 = 0.5948V_S \text{ m}^3 \quad (37)$$

$$KT_3 = 923\{1.76 \times 10^{-3}\}V_S \text{ m}^3 = 1.6245V_S \text{ m}^3 \quad (38)$$

(4) In the isothermal compression process 1-2, the power piston compresses the working fluid from State 1, $p_1 = 1.013 \times 10^5 \text{ N/m}^2$ and $V_{C1} = 0.125 \text{ m}^3$, to State 2, $V_{C2} = 0.075 \text{ m}^3$. At State 1, as the total working fluid is contained only in cold space and dead space, the mass of the working fluid at State 1 is then calculated from Eq. (8):

$$\begin{aligned} m &= (p/R)(V_H/T_3 + K + V_C/T_1) = [(1.013 \times 10^5)/(287)](0 + 1.76 \times 10^{-3} V_S \\ &\quad + 0.125/338) \text{ kg} = 352.9617(1.76 \times 10^{-3} V_S + 369.823 \times 10^{-6}) \text{ kg} \\ &= 0.1305 + 0.6212V_S \text{ kg} \end{aligned} \quad (39)$$

(5) From Eq. (10), the compression work 1-2 is:

$$\begin{aligned} W_{1-2} &= mRT_1 \ln[(V_D + KT_1)/(V_D + V_P + KT_1)] = (0.1305 \\ &\quad + 0.6212V_S)(287)(338) \ln[(0.075 + 0.5948V_S)/(0.125 + 0.5948V_S)] \text{ J} \end{aligned} \quad (40)$$

(6) From Eq. (14), the expansion work 3-4 is:

$$\begin{aligned} W_{3-4} &= Q_{3-4} = mRT_3 \ln[(V_D + V_P + KT_3)/(V_D + KT_3)] = (0.1305 \\ &\quad + 0.6212V_S)(287)(923) \ln[(0.125 + 1.6245V_S)/(0.075 + 1.6245V_S)] \text{ J} \end{aligned} \quad (41)$$

(7) From Eq. (13), the heat added from an external source during process 3'-3 is:

$$\begin{aligned} Q_{3-3'} &= (1 - e)mC_v(T_3 - T_1) \\ &= (1 - e)(0.1305 + 0.6212 V_S)(718)(585) \text{ J} \\ &= (1 - e)(420,030)(0.1305 + 0.6212V_S) \text{ J} \end{aligned} \quad (42)$$

Since the engine network and mean effective pressure have the same characteristics, then only the engine network is studied. The compression work 1-2, the expansion work 3-4, and heat added from an external source during process 3'-3 can be determined by substituting a given regenerator effectiveness and total dead volume into Eqs. (36)–(42). After that, the cycle network, total heat input from an external heat source, and thermal efficiency can be determined from Eqs. (24), (18) and (29), respectively.

4. Results and discussion

Effects of dead volume and regenerator effectiveness on the compression work, expansion work, engine net work, total heat input, and thermal efficiency at various total dead volume ratios ($k_{ST}=0, 0.2, 0.3333, 0.5, 0.6, 0.6667$ or $k_{SDP}=0, 0.25, 0.5, 1, 1.5, 2$) and various regenerator effectiveness values ($e=0, 0.2, 0.4, 0.6, 0.8, 1$) are shown in Figs. 3–6. In these figures, the engine performances are shown in dimensionless form. The additional subscript * represents the condition at zero dead volume and 100% regenerator effectiveness.

Fig. 3 shows the relationship between working fluid mass and total dead volume ratio. Since the arithmetic-mean regenerator effective temperature is not affected by the regenerator effectiveness, then the working fluid mass depends only on dead volume. It can be seen from this figure that the working fluid mass increases with increasing dead volume.

Fig. 4 shows the variations of compression work, expansion work, and engine net work with total dead volume ratio. The characteristics of these curves are similar and independent of the regenerator effectiveness. The compression work, expansion work, and engine network decrease with increasing dead volume. An engine with a large dead

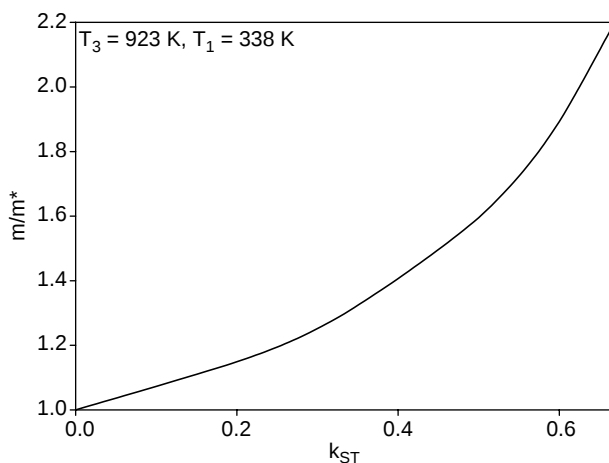


Fig. 3. Working fluid mass against total dead volume ratio.

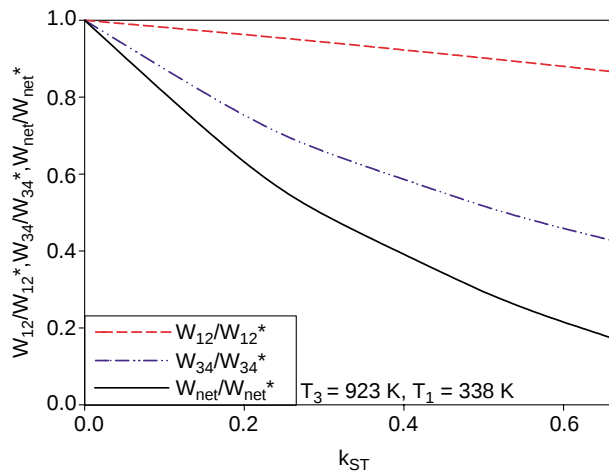


Fig. 4. Compression work, expansion work, and net work against total dead volume ratio.

volume requires a small amount of compression work and gives a small amount of expansion work. Even in the case of an engine of large dead volume, a small amount of engine network can be produced. An engine with a smaller dead volume gives more engine network than an engine with a larger dead volume.

Fig. 5 shows the variations of the total heat input with total dead volume ratio and regenerator effectiveness. The total heat input is the sum of the expansion work and the external heat input. For a perfect regenerator, the external heat input and output is not required and the total heat input decreases with increasing dead volume. An imperfect regenerator requires external heat input, however, the total heat input is likely to increase with increasing dead volume and decreasing regenerator effectiveness. For low

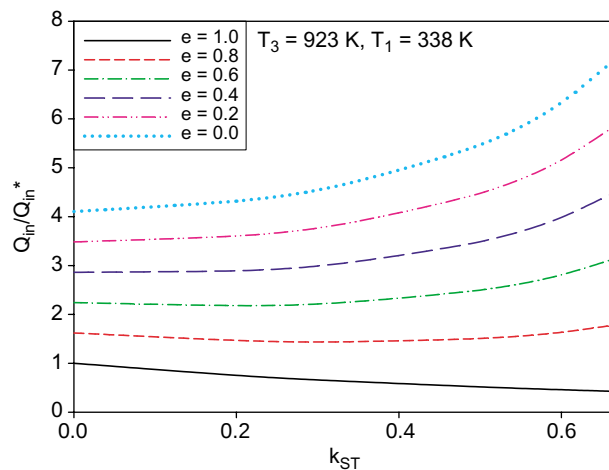


Fig. 5. Total heat input against total dead volume ratio.

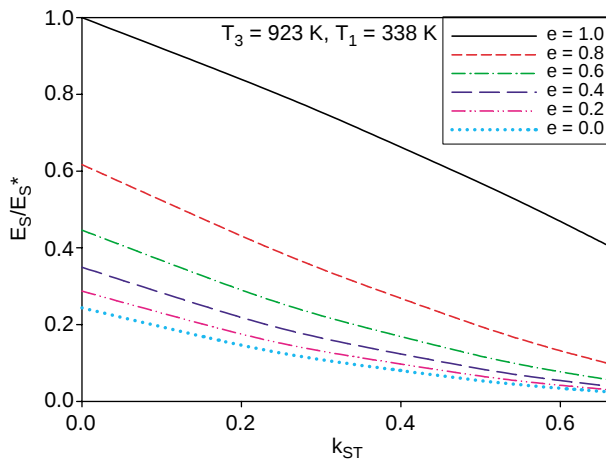


Fig. 6. Thermal efficiency against total dead volume ratio.

regenerator effectiveness values, the total heat input sharply increases with increasing dead volume. An engine with lower regenerator effectiveness requires more total heat input than that with a higher one. An engine with a large dead volume requires a large amount of total heat input.

Effects of dead volume and regenerator effectiveness on thermal efficiency are shown in Fig. 6. The thermal efficiency increases with increasing regenerator effectiveness and decreasing dead volume. An engine with an efficient regenerator and small dead volume will give a high efficiency. It should be noted that a low thermal efficiency can be obtained from a Stirling engine even when it has no regenerator and has a large dead volume.

5. Conclusions

The thermodynamic analysis for an imperfect regeneration Stirling engine with dead volumes is presented in this article. This study shows that an imperfect regeneration Stirling engine with dead volumes and an arithmetic-mean regenerator effective temperature can be analyzed by using basic classical thermodynamics. The analysis presented provides a more generalized and more realistic analytical method for Stirling engine performance evaluation and improvement.

Results from this study indicate that, for an imperfect regeneration Stirling engine with dead volumes and an arithmetic-mean regenerator effective temperature, elements of the engine performance are affected only by the dead volume and others are affected by both the dead volume and the regenerator effectiveness. From this study, we can conclude that:

- (1) For a Stirling engine with a given dead volume, an inefficient regenerator will not affect the engine network if the regenerator effective temperature is an arithmetic

mean of the heater and the cooler temperature. However, an engine with an inefficient regenerator needs more heat input and better cooling than an efficient one.

- (2) The dead volume will decrease both the engine network and the thermal efficiency and will increase both the external heat input and output. However, a real engine must have some unavoidable dead volume.
- (3) A small amount of engine network can be produced even when the engine has a large dead volume.
- (4) To attain high efficiency, a good regenerator is needed. However, the Stirling engine is able to attain a low efficiency without a regenerator.

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